NONEQUILIBRIUM-IONIZED VISCOUS IMPACT LAYER NEAR THE STAGNATION POINT

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The article deals with a hypersonic stream of viscous nonequilibrium-ionized gas around blunt bodies.

This study is concerned with a viscous stream of nonequilibrium-ionized argon in which the following reaction occurs [1]:

$$A + e \rightleftarrows A^* + e, \quad A^* + e \rightleftarrows A^+ + 2e. \tag{1}$$

The main object here is to analyze the effect of relaxation processes and of the transfer coefficients, to various approximations, on the flow field of the impact layer and on the thermal flux at the surface of an immersed body.

For the solution of this problem, we write the fundamental equations in a system of coordinates referred to the body and then perform a transformation in accordance with the well known model of a thin impact layer [2].

Disregarding any transverse pressure variation and taking into account the ambipolarity of the diffusion process, we obtain

$$\frac{\partial}{\partial x}(r\rho u) + \frac{\partial}{\partial y}(r\rho v) = 0, \tag{2}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \tag{3}$$

$$\rho u \frac{\partial \alpha}{\partial x} + \rho v \frac{\partial \alpha}{\partial y} = \frac{\partial}{\partial y}\left(\rho D_A \frac{\partial \alpha}{\partial y}\right) + m_a \dot{n}_{ea}, \tag{4}$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\rho D_A \frac{\partial h}{\partial \alpha}\frac{\partial \alpha}{\partial y}\right) + u\frac{dp}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{5}$$

$$p = \rho RT(1+\alpha). \tag{6}$$

According to [1],

$$\dot{n}_{ea} = \alpha(1-\alpha)\left(\frac{\rho}{m_a}\right)^2 2C_E \sqrt{\frac{2}{\pi m_e}}(KT)^{3/2}\left(\frac{T_x}{T}+2\right)\exp\left(-\frac{T_x}{T}\right)\left[1-\frac{\alpha^2}{1-\alpha}\frac{1-\alpha_E}{\alpha_E^2}\right], \tag{7}$$

where $C_E = 4.4 \cdot 10^{-3}$ m^2/J [3].

The Rankine–Hugoniot relations, supplemented by a constraint on the degree of ionization ($\alpha_\infty = \alpha_S$), serve as the boundary conditions on the shock wave. On the body we have

$$u_w = 0, \quad v_w = 0, \quad T_w = \text{const}, \quad \alpha_w = \alpha_E(T_w, p). \tag{8}$$

We now consider the flow near the stagnation point on the front surface. It is assumed here that all dependent variables in expressions (9)

$$u = u_1(y)x, \quad p = \rho_\infty V_\infty^2(1-\varkappa)\left[1-(x/L)^2\right], \tag{9}$$

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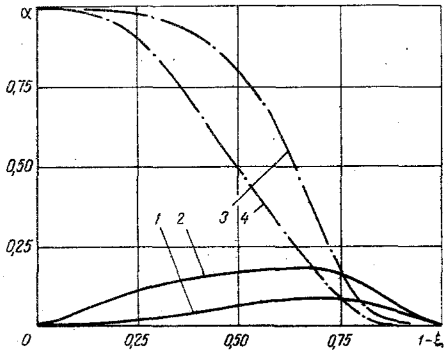

Fig. 1. Profiles of nonequilibrium ionization (curves 1, 2) and of equilibrium ionization (curves 3, 4) across the impact layer, at $Ma_\infty = 15$ (curves 1, 3) and at $Ma_\infty = 17$ (curves 2, 4); α and ξ are dimensionless quantities.

except the tangential components of velocity u and pressure p, are functions of one variable: y. System (2)-(6) is then reduced to a system of ordinary differential equations.

We introduce a variable η and express u_1 as well as v in terms of $f(\eta)$

$$\eta = \sqrt{\frac{V_\infty}{L\rho_s\mu_s}} \int_0^y \rho dy, \quad u_1 = \frac{1}{2}\frac{V_\infty}{L} f'(\eta), \quad v = -\frac{V_\infty}{L} f(\eta) \frac{dy}{d\eta}. \tag{10}$$

We then change to dimensionless variables

$$\bar{v} = \frac{v}{V_\infty}, \quad \bar{\rho} = \frac{\rho}{\rho_\infty}, \quad \bar{p} = \frac{p}{\rho_\infty V_\infty^2}; \quad \bar{T} = \frac{RT}{V_\infty^2}. \tag{11}$$

The final fundamental system of equations for the vicinity of the stagnation point becomes

$$\frac{1}{\eta_s^2}\frac{d}{d\xi}\left(l\frac{d\varphi}{d\xi}\right) + \tilde{\varphi}\frac{d\varphi}{d\xi} - \frac{1}{2}\varphi^2 + \frac{4}{\rho}(1-\varkappa) = 0, \tag{12}$$

$$\frac{1}{\eta_s^2}\frac{d}{d\xi}\left(\frac{l}{\mathrm{Sc}}\frac{d\alpha}{d\xi}\right) + \tilde{\varphi}\frac{d\alpha}{d\xi} + \frac{m_a n_{ea}}{\rho} = 0, \tag{13}$$

$$\frac{1}{\eta_s^2}\frac{d}{d\xi}\left(\frac{l}{\mathrm{Pr}}\frac{dT}{d\xi}\right) + (1+\alpha)\tilde{\varphi}\frac{dT}{d\xi} + \frac{1}{\eta_s^2}\frac{l}{\mathrm{Sc}}\frac{d\alpha}{d\xi}\frac{dT}{d\xi} - \left(T + \frac{2}{5}T_j\right)\frac{m_a n_{ea}}{\rho} = 0 \tag{14}$$

with the following boundary conditions:

$$\xi = 0 \quad \varphi = 0, \ \alpha = \alpha_w, \ T = T_w, \tag{15}$$

$$\xi = 1 \quad \varphi = 2, \ \alpha = \alpha_s, \ T = T_s. \tag{16}$$

Here

$$\xi = \eta/\eta_s, \quad \varphi = \frac{df}{d\eta}, \quad \tilde{\varphi}(\xi) = \int_0^\xi \varphi d\xi, \tag{17}$$

$$l = \rho\mu/\rho_s\mu_s, \quad \mathrm{Pr} = 5R\mu/2\lambda, \quad \mathrm{Sc} = \mu/\rho D_A. \tag{18}$$

In order to solve this system of equations, we must know l, Pr, and Sc as functions of the thermodynamic properties of the gas. For calculating the transfer coefficients which appear in (18), one usually expands the term added to the equilibrium distribution function into a Sonin polynomial series [4, 5]. Studies have shown that, for an analysis of ionized hot gases it is necessary that the dynamic viscosity μ be given to the second approximation and the thermal conductivity λ be given to the fourth approximation. Meanwhile, both μ_2 and λ_4 have been calculated only for quiescent gases at equilibrium (for argon [4, 5] and for air [6]) without any consideration given to the effects of μ_2 and λ_4 on the gas-dynamic aspects of the flow.

In our study here μ and λ have been calculated to these higher-order approximations for nonequilibrium flow and system (12)-(14) has been solved with the thus more precise values of the transfer coefficients, but also with these coefficients based on the simpler classical theory [7] and for the case $l = 1$, $\mathrm{Pr} = 2/3$, and $\mathrm{Sc} = 1$. For calculating μ_1, μ_2, λ_1, and λ_4 we used the interaction potentials for partially ionized argon according to [5].

The results of these calculations are shown in Figs. 1 and 2 for L = 0.04 m and $T_W = 2000°K$, for example, with the parameters of the oncoming stream $Ma_\infty = 15$ and 17 respectively, $p_\infty = 100\ N/m^2$, and $\alpha_\infty = 10^{-3}$.

TABLE 1. Thermal Flux at the Wall Surface, $q_W \cdot 10^{-3}\ W/m^2$

Ma_∞	$q_W \cdot 10^{-3}$			
	calculated with $l = 1$, Pr = 2/3, and Sc = 1	with the transfer coefficients according to [7]	with μ_1 and λ_2	with μ_2 and λ_4
14	3386	4239	4333	4592
15	3962	4739	5254	5825
16	4692	5211	5978	6615
17	5595	5941	6933	7619
18	6718	6933	8216	8992

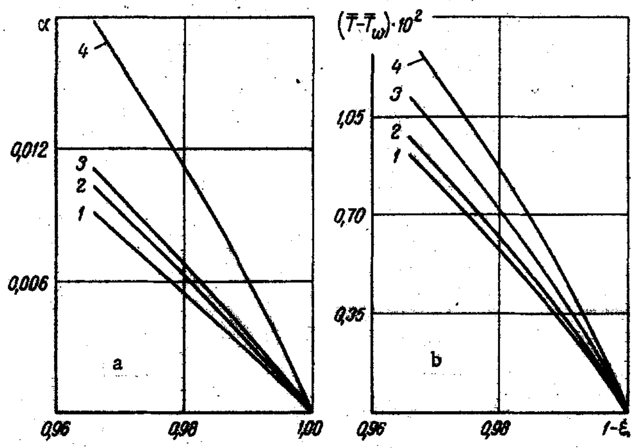

Fig. 2. Profiles of the ionization (a) and of the temperature (b) across the boundary layer, at $Ma_\infty = 17$: 1) calculated with transfer coefficients according to the classical theory [7], 2) with μ_1 and λ_2, 3) with μ_2 and λ_4, 4) with $l = 1$, $Pr = 2/3$, and $Sc = 1$; α, $\overline{T}-\overline{T}_W$; and ξ are dimensionless quantities.

The solution was obtained by the elimination method with iterations performed on a model BÉSM-4 digital computer. In order to save computer time, tables of transfer coefficients were stored in the memory which had been calculated beforehand to various approximations and as functions of the thermodynamic properties.

In Fig. 1 are shown profiles of the nonequilibrium ionization (α) and of the equilibrium ionization (α_E) across the impact layer. Obviously, both profiles have quite different trends.

In Fig. 2 are shown profiles of the nonequilibrum ionization α and of the temperature $\overline{T}$ across the boundary layer, with the transfer coefficients calculated to various approximations. It is noteworthy that, as the transfer coefficients become more precise, α and $\overline{T}$ seem to increase in the boundary layer, while calculations with $l = 1$, $Pr = 2/3$, and $Sc = 1$ yield much too high values for α and $\overline{T}$.

In Table 1 are given the values of the thermal flux at the wall surface

$$q_w = -\lambda \frac{dT}{dy}\Big|_w - \rho D_A \left(\frac{5}{2} RT + RT_j \right) \frac{d\alpha}{dy}\Big|_w \tag{19}$$

as a function of the Mach number Ma_∞, and of the transfer coefficients calculated to various approximations. Obviously, more precise values of the transfer coefficients yield higher values for the thermal flux at the wall.

NOTATION

A	is the atom in its fundamental state;
A^*	is the atom in an excited state;
A^+	is the single-charge ion;
x, y	are the space coordinates;
u, v	are the velocity components along the x, y axes respectively;
ξ	is the dimensionless transverse coordinate across the impact layer;
V_∞	is the gas velocity in the oncoming stream;
p	is the gas pressure;
ρ	is the gas density;
T	is the gas temperature;
α	is the degree of gas ionization;
α_E	is the degree of equilibrium ionization;
$h = 5RT(1 + \alpha)/2 + \alpha RT_j$	is the specific enthalpy of the mixture;
R	is the specific gas constant;
T_j	is the ionization temperature;

T_X	is the excitation temperature;
K	is the Boltzmann constant;
m_a	is the mass of an atom;
m_e	is the mass of an electron;
$\dot{n}_{ea}$	is the rate of the ionization reaction;
$\varkappa$	is the ratio of densities before and just behind the shock wave;
μ	is the dynamic viscosity;
λ	is the thermal conductivity;
DA	is the ambipolar diffusivity;
μ_1, μ_2	are the dynamic viscosity to the first and to the second approximation respectively;
λ_2, λ_4	are the thermal conductivity to the second and to the fourth approximation respectively;
Ma	is the Mach number;
Pr	is the Prandtl number;
Sc	is the Schmidt number;
$l = \rho\mu/\rho_s\mu_s$	is a dimensionless parameter;
q	is the thermal flux.

Subscripts

∞	denotes the gas in the oncoming stream;
s	denotes the conditions just behind the shock wave;
w	denotes the body surface.

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